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Progress Report

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A calculation has been made of the spatial distribution of absorbed dose in a water medium around monoenergetic point-isotropic electron sources. The calculation takes into account angular deflections and energy-loss straggling due to multiple Coulomb scattering by atoms and orbital electrons; it also includes the transport of energy by secondary bremsstrahlung. The results are presented in the form of scaled point kernels for 36 source energies between 10 MeV and 0.5 keV. The scaling is done by expressing all distances in units of the electron mean range, and makes possible easy interpolation to any source energy in the interval covered. In order to illustrate the use of the point kernels, applications are made to two problems arising in beta-ray dosimetry. The first problem pertains to the self-absorption of energy in spherical source regions. The other problem concerns the absorbed-dose distribution as a function of the distance from a semi-infinite uniform source region.

1. INTRODUCTION

This is a preliminary report dealing with the calculation of electron point kernels in water. These kernels are functions which represent the distribution of absorbed dose around point-isotropic sources of electrons. The purpose of the report is to provide interim documentation for other papers on electron and beta-ray dosimetry which make use of these point kernels. Work on a detailed description of the calculation is in progress.

An extensive and systematic set of data on the distribution of absorbed dose around point-isotropic electron sources is contained in the tabulation of Spencer,[1/] whose results for a carbon medium have been used, with a slight scaling adjustment, to produce point kernels for a water medium, both for monoenergetic sources and for a large number of radionuclide beta-ray sources.[2/]

Spencer's calculations used the continuous-slowing-down approximation, i.e. a schematization in which the rate of energy loss at each point along an electron trajectory is assumed to be equal to the mean energy loss given by the Bethe stopping power formula. Energy-loss straggling is disregarded in this approximation, which has the result that the amount of absorbed dose is slightly overestimated near the source and underestimated far from the source.

The calculations reported here are an improvement in the following respects:

(1) Energy-loss straggling is taken into account, both for energy losses suffered in collisions with atomic electrons and for bremsstrahlung losses.

(2) The effect of energy transport by secondary bremsstrahlung on the point kernel functions is included.

(3) The calculations are extended to a low-energy region not previously covered (25 keV to 0.5 keV).

2. SCALED POINT KERNEL

The distribution of absorbed dose in an unbounded medium, around a point-isotropic source emitting electrons of energy E_o, can be expressed in terms of

the specific absorbed fraction $\Phi(r,E_o)$. This quantity is defined to be the fraction of the emitted energy that is absorbed per unit mass of the medium at a distance r from the point source. In order to minimize the dependence on E_o and thus to facilitate interpolation, it is convenient to express the specific absorbed fraction in terms of a scaled point kernel $F(r/r_o,E_o)$ defined by the equation

$$F(r/r_o,E_o)d(r/r_o) = 4\pi\rho\Phi(r,E_o)r^2dr, \qquad (1)$$

where r_o is the c.s.d.a.[*] range at energy E_o and ρ is the density of the medium. The scaling is accomplished by expressing distances in units of r_o.

Table 1 gives the scaled point kernel $F(r/r_o,E_o)$ for a water medium, for 36 values of the source energy E_o between 10 MeV and 0.5 keV. The results for $E_o > 20$ keV were obtained by a variant[3,4] of the Monte Carlo method which combined random sampling with the use of the Bethe stopping power theory as formulated by Rohrlich and Carlson,[5] the Landau[6] energy-loss straggling distribution with the Blunck-Leisegang[7] binding correction, and the Goudsmit-Saunderson[8] multiple-scattering angular distribution evaluated with use of the Mott[9] elastic scattering cross section and the Molière[10,11] screening correction. For $E_o \leq 20$ keV another Monte Carlo model[12] was used in which all elastic collisions with atoms and hard inelastic collisions with atomic electrons were followed by random sampling. The numerous soft inelastic collisions (giving rise to atomic excitation or to the production of knock-on electrons with energies smaller than 200 eV) were treated in the continuous-slowing-down approximation, with the use of a "restricted" stopping power. The stopping power values at energies below 10 keV were taken from a semi-empirical formula of Green[13,14] and collaborators which gives results that are in reasonable agreement with experimental

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* Continuous-slowing-down approximation.

values of Cole.[15,16]

Table 2 gives, for all 36 source energies, the values of the c.s.d.a. range r_o and the values of the 90-percentile distance x_{90}. The 90-percentile distance is defined to be the radius of the sphere around a point source within which 90% of the emitted energy is absorbed. Table 3 compares, for source energies of 1 MeV and 0.1 MeV, the new scaled point kernel (with straggling effects) and the old scaled point kernel (without straggling effects). Table 4 compares the 90-percentile distances obtained with and without inclusion of straggling effects.

3. ABSORPTION OF ENERGY IN A SPHERICAL SOURCE REGION

It may happen, either as the result of deliberate tagging procedures[17,18] in radiobiological experiments, or as the result of accidental ingestion of radioactive material, that radionuclides get incorporated into the genetic material of the cell, particularly the cell nucleus. It is then of interest to know what fraction of the emitted beta-ray and electron energy is absorbed within the cell nucleus, and what fraction is absorbed in the surrounding - and presumably less sensitive - material. This question can be answered with use of the point kernel tabulated in Table 1.

Sample calculations of this problem have been done for the case that the cell nucleus can be represented as a spherical source region with a diameter of a few microns. The fraction $A(d,E_o)$ of the emitted energy absorbed in a sphere of diameter d can be obtained as an integral over the point kernel. It can be shown that

$$A(d,E_o) = \frac{1}{r_o} \int_o^d \left[1 - 1.5\ (r/d) + 0.5\ (r/d)^3\right] F(r/r_o,E_o)dr. \qquad (2)$$

The fraction $A(d,E_o)$ is tabulated in Table 5 for source energies between 10 MeV and 0.5 keV and for sphere diameters between 2 and 14 μm. If the source is

characterized by a spectrum $S(E_o)$ (continuous for beta-rays, discrete for conversion and Auger electrons), then the average fraction of the energy absorbed in the source region is

$$A_{av}(d) = \frac{1}{E_{av}} \int_o^\infty S(E_o)\, E_o\, A(d,E_o) dE_o, \tag{3}$$

where

$$E_{av} = \int_o^\infty S(E_o) E_o dE_o \tag{4}$$

is the average energy of the emitted particles.

The average absorbed fraction $A_{av}(d)$ has recently been calculated by Ertl[19] for the radionuclides tritium and iodine-125. He assumed straight-line motion of the electrons away from the point-isotropic source, and an effective rate of energy deposition per unit pathlength in accordance with an experimental range-energy relation. The relation used was one determined by Cole[15] in a study of the penetration of low-energy electrons through collodion foils.

In the present work, the beta spectra and discrete electron spectra were taken from the work of Martin and Blichert-Toft[20] and Gove and Martin,[21] except for iodine-125, whose discrete electron spectrum was taken from Ertl.[19] The values of $A_{av}(d)$ thus obtained are given in Table 6 for a few radionuclides. In particular, a comparison is made with Ertl's results. It can be seen that agreement is close in the case of tritium, but not so good in the case of iodine-125.

4. REDUCTION FACTOR FOR A HALF-SPACE SOURCE

Another application of the tabulated point kernel has been made in order to determine the depth-dose distribution in a (tissue-equivalent) water target exposed to electron and beta radiation from a surrounding radioactive cloud.[22]

Such a cloud can arise as the result of the injection of reactor effluents into the atmosphere, or as the result of a nuclear accident.

As shown in Ref. 22, the solution of this problem can be accomplished in two steps: (a) First it is assumed that a radioactive source is distributed uniformly through one half-space of a water medium, and one calculates the depth-dose distribution in the other half space. (b) One then takes into account that the radioactive source is distributed in air rather than water, and makes the appropriate corrections to account for the differences of the scattering properties of the two media.

Step (a) involves the calculation of a half-space reduction factor for mono-energetic sources. Let R_o be the absorbed-dose rate that would prevail everywhere in an unbounded homogeneous medium if a uniform isotropic source emitting electrons of energy E_o were distributed throughout the entire medium. Let $R(z,E_o)$ be the corresponding absorbed-dose rate at a depth $z > 0$ that would prevail if the source were confined to the half-space $z < 0$. The <u>reduction factor</u>, i.e. the ratio $G = R/R_o$, can be expressed as follows in terms of the scaled point-kernel:

$$G(z/r_o, E_o) = \frac{1}{2} \int_{z/r_o}^{\infty} [1 - (z/r_o)/t] \, F(t, E_o) dt. \tag{5}$$

Values of the reduction factor for a water medium are given in the attached Table 7.

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r/r_o	E_o, MeV					
	.0005	.0006	.0008	.0010	.0015	.0020
.00	.711	.673	.632	.607	.570	.565
.05	.829	.788	.744	.711	.674	.645
.10	.942	.900	.850	.820	.770	.744
.15	1.062	1.015	.964	.924	.869	.836
.20	1.196	1.140	1.074	1.026	.963	.926
.25	1.335	1.264	1.190	1.142	1.067	1.024
.30	1.486	1.399	1.312	1.259	1.181	1.138
.35	1.603	1.521	1.438	1.383	1.302	1.253
.40	1.685	1.604	1.526	1.475	1.400	1.355
.45	1.723	1.652	1.586	1.539	1.474	1.435
.50	1.703	1.644	1.596	1.563	1.520	1.493
.55	1.625	1.592	1.566	1.543	1.520	1.507
.60	1.408	1.463	1.474	1.479	1.482	1.485
.65	1.212	1.262	1.324	1.353	1.396	1.415
.70	.877	.995	1.126	1.192	1.254	1.277
.75	.535	.756	.886	.962	1.029	1.062
.80	.254	.446	.506	.685	.776	.832
.85	.090	.177	.312	.403	.509	.593
.90	.016	.046	.114	.184	.309	.377
.95	.000	.000	.000	.050	.160	.210
1.00	.000	.000	.000	.002	.058	.090
1.05	.000	.000	.000	.000	.000	.020
1.10	.000	.000	.000	.000	.000	.000
1.15	.000	.000	.000	.000	.000	.000
1.20	.000	.000	.000	.000	.000	.000
r/r_o	.0030	.0040	.0050	.0060	.0080	.0100
.00	.533	.524	.518	.514	.510	.508
.05	.625	.613	.608	.598	.590	.589
.10	.716	.693	.586	.679	.669	.663
.15	.796	.775	.762	.755	.743	.733
.20	.888	.868	.847	.837	.817	.803
.25	.982	.942	.939	.928	.908	.894
.30	1.091	1.065	1.043	1.030	1.008	.990
.35	1.203	1.159	1.154	1.129	1.103	1.084
.40	1.305	1.271	1.250	1.229	1.207	1.195
.45	1.395	1.374	1.354	1.343	1.319	1.305
.50	1.464	1.448	1.433	1.422	1.410	1.399
.55	1.492	1.482	1.477	1.476	1.474	1.472
.60	1.486	1.485	1.489	1.492	1.498	1.504
.65	1.424	1.432	1.437	1.448	1.462	1.476
.70	1.301	1.322	1.338	1.351	1.376	1.395
.75	1.125	1.161	1.164	1.203	1.229	1.241
.80	.905	.952	.979	1.000	1.028	1.049
.85	.662	.709	.738	.757	.783	.803
.90	.437	.472	.494	.512	.534	.548
.95	.261	.265	.295	.305	.321	.329
1.00	.120	.139	.152	.161	.173	.179
1.05	.044	.056	.062	.065	.068	.070
1.10	.003	.016	.020	.022	.024	.024
1.15	.000	.000	.000	.000	.000	.000
1.20	.000	.000	.000	.000	.000	.000

Table 1. Scaled point kernel $F(r/r_o, E_o)$ in water.

r/r_o	E_o, MeV .0150	.0200	.0300	.0400	.0500	.0600
.00	.508	.510	.521	.528	.536	.544
.05	.582	.576	.579	.532	.585	.592
.10	.654	.648	.643	.640	.640	.640
.15	.717	.705	.696	.690	.690	.688
.20	.781	.769	.754	.746	.740	.740
.25	.869	.855	.834	.820	.814	.810
.30	.967	.954	.932	.920	.912	.906
.35	1.062	1.050	1.032	1.022	1.010	1.008
.40	1.172	1.162	1.151	1.145	1.140	1.134
.45	1.281	1.271	1.273	1.285	1.290	1.292
.50	1.391	1.389	1.403	1.417	1.424	1.428
.55	1.475	1.479	1.505	1.525	1.550	1.556
.60	1.519	1.540	1.578	1.607	1.622	1.628
.65	1.503	1.528	1.582	1.609	1.624	1.634
.70	1.425	1.449	1.479	1.501	1.516	1.528
.75	1.274	1.299	1.315	1.323	1.328	1.336
.80	1.074	1.090	1.090	1.084	1.080	1.076
.85	.819	.815	.796	.780	.768	.764
.90	.558	.552	.523	.506	.492	.484
.95	.339	.335	.313	.296	.274	.264
1.00	.185	.181	.164	.148	.134	.126
1.05	.072	.073	.074	.072	.070	.066
1.10	.023	.021	.019	.016	.015	.013
1.15	.000	.000	.000	.000	.000	.000
1.20	.000	.000	.000	.000	.000	.000

r/r_o	.0800	.1000	.1500	.2000	.3000	.4000
.00	.559	.572	.595	.611	.640	.660
.05	.603	.612	.633	.648	.674	.690
.10	.645	.643	.661	.676	.702	.725
.15	.691	.692	.703	.716	.745	.755
.20	.741	.744	.757	.770	.791	.807
.25	.809	.803	.820	.828	.845	.855
.30	.901	.899	.898	.903	.908	.912
.35	1.005	1.003	.998	.997	.996	.994
.40	1.131	1.131	1.123	1.108	1.113	1.109
.45	1.294	1.296	1.292	1.283	1.268	1.256
.50	1.430	1.432	1.426	1.428	1.405	1.395
.55	1.562	1.561	1.557	1.535	1.522	1.507
.60	1.638	1.645	1.643	1.639	1.622	1.604
.65	1.644	1.643	1.653	1.651	1.638	1.622
.70	1.546	1.557	1.563	1.558	1.544	1.534
.75	1.342	1.348	1.354	1.359	1.361	1.358
.80	1.073	1.073	1.073	1.072	1.071	1.071
.85	.757	.752	.743	.740	.735	.735
.90	.469	.461	.446	.438	.429	.425
.95	.246	.237	.219	.213	.193	.182
1.00	.114	.100	.086	.076	.066	.060
1.05	.055	.042	.026	.020	.014	.012
1.10	.011	.009	.007	.005	.003	.002
1.15	.000	.000	.000	.000	.000	.000
1.20	.000	.000	.000	.000	.000	.000

Table 1. Continued.

r/r_o	E_o, MeV					
	.5000	.6000	.8000	1.0000	1.5000	2.0000
.00	.674	.689	.714	.736	.771	.798
.05	.708	.727	.752	.774	.811	.834
.10	.744	.757	.784	.807	.845	.866
.15	.784	.795	.820	.835	.871	.894
.20	.821	.832	.860	.867	.897	.920
.25	.865	.876	.892	.905	.935	.952
.30	.919	.924	.936	.949	.969	.980
.35	.995	1.010	1.012	1.003	1.007	1.010
.40	1.104	1.099	1.088	1.083	1.071	1.062
.45	1.247	1.231	1.211	1.196	1.157	1.134
.50	1.384	1.372	1.355	1.338	1.285	1.232
.55	1.498	1.484	1.457	1.435	1.380	1.325
.60	1.579	1.563	1.529	1.501	1.438	1.393
.65	1.605	1.587	1.555	1.529	1.472	1.423
.70	1.522	1.510	1.489	1.475	1.440	1.413
.75	1.351	1.346	1.339	1.334	1.319	1.313
.80	1.070	1.067	1.064	1.067	1.077	1.088
.85	.736	.739	.742	.748	.765	.782
.90	.422	.424	.427	.431	.455	.481
.95	.185	.183	.184	.191	.206	.230
1.00	.058	.056	.056	.058	.066	.078
1.05	.010	.010	.009	.008	.008	.013
1.10	.002	.002	.001	.001	.001	.002
1.15	.000	.000	.000	.000	.000	.000
1.20	.000	.000	.000	.000	.000	.000
r/r_o	3.0000	4.0000	5.0000	6.0000	8.0000	10.0000
.00	.826	.835	.841	.841	.839	.833
.05	.856	.868	.875	.877	.873	.867
.10	.891	.900	.905	.906	.903	.897
.15	.913	.924	.929	.930	.925	.917
.20	.941	.950	.949	.948	.941	.936
.25	.967	.970	.967	.966	.957	.948
.30	.979	.978	.976	.972	.963	.958
.35	1.007	1.002	.996	.992	.980	.964
.40	1.043	1.028	1.010	1.000	.986	.976
.45	1.093	1.066	1.046	1.032	1.010	.989
.50	1.161	1.119	1.090	1.065	1.032	1.004
.55	1.248	1.193	1.149	1.119	1.066	1.024
.60	1.318	1.259	1.211	1.173	1.110	1.054
.65	1.360	1.303	1.257	1.217	1.147	1.086
.70	1.358	1.313	1.275	1.242	1.179	1.124
.75	1.290	1.265	1.249	1.226	1.181	1.136
.80	1.111	1.125	1.126	1.131	1.130	1.102
.85	.842	.900	.945	.962	.986	.998
.90	.552	.609	.654	.694	.756	.803
.95	.273	.311	.352	.388	.465	.542
1.00	.104	.129	.155	.181	.235	.295
1.05	.020	.028	.036	.045	.067	.098
1.10	.003	.005	.008	.012	.018	.025
1.15	.001	.001	.002	.003	.006	.008
1.20	.000	.000	.000	.000	.001	.003

Table 1. Continued.

E_o (MeV)	r_o (g/cm^2)	x_{90}/r_o	x_{90} (g/cm^2)
.0005	2.272-06	.66483	1.510-06
.0006	2.897-06	.69605	2.016-06
.0008	4.325-06	.72139	3.120-06
.0010	5.976-06	.73969	4.420-06
.0015	1.092-05	.76543	8.358-06
.0020	1.710-05	.78250	1.338-05
.0030	3.279-05	.79916	2.620-05
.0040	5.268-05	.80849	4.259-05
.0050	7.652-05	.81225	6.215-05
.0060	1.037-04	.81581	8.460-05
.0080	1.689-04	.82035	1.386-04
.0100	2.482-04	.82328	2.043-04
.0150	5.042-04	.82568	4.163-04
.0200	8.374-04	.82482	6.907-04
.0300	1.715-03	.82016	1.407-03
.0400	2.851-03	.81626	2.327-03
.0500	4.222-03	.81326	3.434-03
.0600	5.807-03	.81152	4.712-03
.0800	9.562-03	.80859	7.732-03
.1000	1.401-02	.80760	1.131-02
.1500	2.760-02	.80416	2.219-02
.2000	4.400-02	.80266	3.532-02
.3000	8.263-02	.80049	6.614-02
.4000	1.264-01	.80011	1.011-01
.5000	1.735-01	.80009	1.388-01
.6000	2.227-01	.80043	1.783-01
.8000	3.248-01	.80155	2.603-01
1.0000	4.297-01	.80341	3.452-01
1.5000	6.956-01	.80731	5.616-01
2.0000	9.613-01	.81334	7.819-01
3.0000	1.485+00	.82839	1.230+00
4.0000	1.997+00	.84244	1.682+00
5.0000	2.499+00	.85551	2.138+00
6.0000	2.991+00	.86458	2.586+00
8.0000	3.950+00	.88849	3.510+00
10.0000	4.880+00	.91118	4.447+00

Table 2. Electron c.s.d.a. range and 90-percentile distance in water.

	$F(r/r_o, E_o)$			
	E_o = 1 MeV		E_o = 0.1 MeV	
r/r_o	c.s.d.a. approx.	with stragg.	c.s.d.a. approx.	with stragg.
0.00	0.807	0.736	0.588	0.572
0.05	0.814	0.774	0.605	0.612
0.10	0.831	0.807	0.639	0.648
0.15	0.859	0.835	0.688	0.692
0.20	0.896	0.867	0.753	0.744
0.25	0.943	0.905	0.833	0.808
0.30	1.001	0.949	0.928	0.899
0.35	1.071	1.003	1.040	1.003
0.40	1.152	1.083	1.166	1.131
0.45	1.247	1.196	1.305	1.296
0.50	1.353	1.338	1.456	1.432
0.55	1.468	1.435	1.610	1.561
0.60	1.573	1.501	1.755	1.645
0.65	1.636	1.529	1.856	1.649
0.70	1.615	1.475	1.833	1.557
0.75	1.450	1.334	1.569	1.348
0.80	1.081	1.067	1.049	1.073
0.85	0.529	0.748	0.484	0.752
0.90	0.078	0.431	0.128	0.461
0.95	0.000	0.191	0.009	0.237
1.00		0.058	0.000	0.100
1.10		0.001		0.009

Table 3. Comparison of point kernels for water calculated with inclusion of energy-loss straggling, or in continuous-slowing-down approximation.

E_o (MeV)	x_{90} (stragg)/x_{90} (c.s.d.a.)
5	1.077
3	1.067
2	1.057
1	1.055
0.5	1.055
0.3	1.056
0.2	1.058
0.1	1.064
0.05	1.074
0.025	1.090

Table 4. Comparison of 90-percentile distances in water computed with inclusion of energy-loss straggling, or in continuous-slowing-down approximation.

E_o (MeV)	SPHERE DIAMETER d (μm) 2.0	4.0	6.0	8.0	10.0	12.0	14.0
.0005	.993825	.997256	.998400	.998971	.999315	.999543	.999707
.0006	.991512	.996061	.997578	.998336	.998791	.999094	.999311
.0008	.986447	.993512	.995867	.997044	.997751	.998222	.998558
.0010	.980446	.990478	.993823	.995495	.996499	.997167	.997645
.0015	.962371	.981447	.987809	.990990	.992898	.994171	.995080
.0020	.939208	.969749	.979941	.985038	.988097	.990136	.991593
.0030	.880713	.940426	.960419	.970426	.976432	.980437	.983298
.0040	.806373	.902684	.935173	.951459	.961241	.967766	.972428
.0050	.719371	.857690	.905001	.928790	.943094	.952640	.959463
.0060	.623808	.806478	.870416	.902723	.922185	.935186	.944482
.0080	.422924	.686869	.768301	.840508	.872177	.893404	.908611
.0100	.258454	.551294	.691103	.765804	.811734	.842718	.864997
.0150	.098152	.248836	.418147	.541654	.625204	.684006	.727262
.0200	.053287	.122803	.213215	.318947	.417707	.497813	.560790
.0300	.024375	.052013	.082989	.118188	.158576	.204532	.255163
.0400	.014420	.029935	.046535	.064208	.083131	.103556	.125726
.0500	.009743	.019939	.030609	.041748	.053344	.065418	.078047
.0600	.007142	.014511	.022109	.029934	.037986	.046267	.054782
.0800	.004421	.008920	.013496	.018148	.022875	.027675	.032551
.1000	.003078	.006189	.009334	.012511	.015720	.018961	.022251
.1500	.001620	.003250	.004888	.006534	.008189	.009852	.011523
.2000	.001044	.002090	.003141	.004194	.005250	.006309	.007372
.3000	.000581	.001164	.001747	.002331	.002916	.003501	.004086
.4000	.000392	.000784	.001176	.001569	.001962	.002355	.002749
.5000	.000291	.000583	.000874	.001166	.001458	.001751	.002043
.6000	.000232	.000464	.000697	.000929	.001162	.001395	.001627
.8000	.000165	.000330	.000495	.000660	.000825	.000990	.001155
1.0000	.000129	.000257	.000386	.000514	.000643	.000772	.000900
1.5000	.000083	.000165	.000249	.000333	.000416	.000499	.000582
2.0000	.000062	.000124	.000187	.000249	.000311	.000373	.000436
3.0000	.000042	.000083	.000125	.000167	.000209	.000250	.000292
4.0000	.000031	.000063	.000094	.000126	.000157	.000188	.000220
5.0000	.000025	.000050	.000076	.000101	.000126	.000151	.000177
6.0000	.000021	.000042	.000063	.000084	.000105	.000127	.000148
8.0000	.000016	.000032	.000048	.000064	.000080	.000096	.000111
10.0000	.000013	.000026	.000038	.000051	.000064	.000077	.000090

Table 5. Fraction $A(d,E_o)$ of the emitted source energy that is absorbed in a spherical source region of diameter d.

d	^{3}H		^{14}C	^{35}S	^{125}I	
	E_{av} = 5.68 keV		E_{av} = 49.3 keV	E_{av} = 48.8 keV	E_{av} = 3.34 keV	
(μm)	Ertl	This work	This work	This work	Ertl	This work
2	50.6	45.7	1.7	1.9	45.3	44.8
4	68.3	65.5	3.4	3.6	56.1	49.7
6	76.0	75.6	4.9	5.1	63.1	53.0
8	81.6	81.4	6.4	6.6	68.3	56.3
10	84.9	85.0	7.8	8.1	71.5	59.5
12	87.5	87.4	9.2	9.4	74.6	62.7
14	89.2	89.2	10.6	10.7	77.1	65.6

Table 6. Percentage of the emitted source energy, 100 $A_{av}(d)$, that is absorbed in a spherical source region of diameter d.

z/r_o	E_o (MeV) .0005	.0006	.0008	.0010	.0015	.0020	.0030	.0040	.0050
.00	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
.05	.3985	.4018	.4052	.4077	.4110	.4131	.4151	.4164	.4170
.10	.3249	.3301	.3355	.3393	.3447	.3480	.3513	.3533	.3544
.15	.2635	.2700	.2767	.2815	.2883	.2924	.2966	.2994	.3007
.20	.2109	.2183	.2259	.2314	.2391	.2438	.2487	.2517	.2534
.25	.1659	.1738	.1819	.1877	.1961	.2011	.2063	.2097	.2113
.30	.1275	.1357	.1439	.1498	.1584	.1635	.1689	.1723	.1741
.35	.0954	.1033	.1114	.1172	.1256	.1307	.1360	.1393	.1411
.40	.0689	.0764	.0840	.0895	.0974	.1023	.1074	.1106	.1123
.45	.0478	.0545	.0613	.0664	.0737	.0782	.0829	.0858	.0874
.50	.0314	.0373	.0431	.0475	.0540	.0580	.0622	.0648	.0663
.55	.0192	.0240	.0289	.0326	.0382	.0416	.0452	.0474	.0487
.60	.0103	.0145	.0182	.0212	.0257	.0286	.0316	.0334	.0345
.65	.0054	.0079	.0106	.0128	.0164	.0187	.0211	.0225	.0234
.70	.0023	.0038	.0055	.0071	.0097	.0115	.0133	.0144	.0150
.75	.0008	.0015	.0024	.0034	.0053	.0066	.0079	.0086	.0090
.80	.0002	.0004	.0008	.0014	.0026	.0034	.0043	.0047	.0050
.85	.0000	.0001	.0002	.0005	.0011	.0016	.0021	.0024	.0025
.90	.0000	.0000	.0000	.0001	.0004	.0006	.0009	.0010	.0011
.95	.0000	.0000	.0000	.0000	.0001	.0002	.0003	.0004	.0004
1.00	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0001
1.05	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 7. Reduction factor $G(z/r_o, E_o)$ for a uniform half-space source in a water medium.

	E_o (MeV)								
z/r_o	.0060	.0080	.0100	.0150	.0200	.0300	.0400	.0500	.0600
.00	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
.05	.4176	.4184	.4188	.4195	.4200	.4201	.4200	.4199	.4197
.10	.3554	.3567	.3575	.3587	.3595	.3598	.3599	.3599	.3597
.15	.3019	.3036	.3047	.3063	.3073	.3079	.3081	.3081	.3079
.20	.2548	.2567	.2580	.2600	.2611	.2618	.2621	.2621	.2620
.25	.2129	.2150	.2165	.2186	.2197	.2205	.2207	.2208	.2207
.30	.1757	.1779	.1793	.1815	.1826	.1833	.1835	.1835	.1835
.35	.1427	.1449	.1464	.1485	.1495	.1501	.1502	.1501	.1500
.40	.1138	.1159	.1173	.1192	.1201	.1205	.1204	.1203	.1202
.45	.0886	.0907	.0919	.0937	.0944	.0946	.0943	.0941	.0939
.50	.0675	.0691	.0702	.0717	.0722	.0722	.0718	.0714	.0712
.55	.0497	.0511	.0520	.0531	.0535	.0532	.0527	.0523	.0521
.60	.0353	.0364	.0371	.0380	.0382	.0377	.0372	.0368	.0365
.65	.0240	.0248	.0253	.0259	.0260	.0255	.0250	.0246	.0243
.70	.0155	.0160	.0164	.0168	.0168	.0163	.0159	.0155	.0153
.75	.0093	.0097	.0100	.0102	.0102	.0098	.0094	.0091	.0089
.80	.0052	.0054	.0056	.0057	.0057	.0054	.0051	.0049	.0048
.85	.0026	.0028	.0029	.0029	.0029	.0027	.0026	.0024	.0023
.90	.0012	.0012	.0013	.0013	.0013	.0012	.0011	.0011	.0010
.95	.0005	.0005	.0005	.0005	.0005	.0005	.0004	.0004	.0004
1.00	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001	.0001
1.05	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 7. Continued

	E_o (MeV)								
z/r_o	.0800	.1000	.1500	.2000	.3000	.4000	.5000	.6000	.8000
.00	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
.05	.4192	.4187	.4177	.4168	.4153	.4143	.4133	.4123	.4109
.10	.3591	.3586	.3573	.3561	.3540	.3526	.3512	.3500	.3480
.15	.3074	.3069	.3054	.3041	.3018	.3002	.2987	.2974	.2952
.20	.2614	.2609	.2594	.2582	.2559	.2544	.2529	.2516	.2494
.25	.2202	.2197	.2183	.2171	.2150	.2136	.2122	.2109	.2089
.30	.1830	.1825	.1812	.1802	.1783	.1771	.1759	.1748	.1731
.35	.1495	.1490	.1479	.1470	.1454	.1444	.1434	.1425	.1411
.40	.1197	.1192	.1182	.1175	.1162	.1154	.1146	.1138	.1126
.45	.0934	.0929	.0920	.0915	.0904	.0898	.0891	.0886	.0879
.50	.0707	.0702	.0694	.0690	.0681	.0677	.0672	.0668	.0664
.55	.0516	.0512	.0504	.0501	.0494	.0490	.0487	.0485	.0482
.60	.0361	.0356	.0349	.0347	.0341	.0339	.0337	.0335	.0334
.65	.0239	.0235	.0229	.0226	.0221	.0220	.0219	.0218	.0218
.70	.0149	.0145	.0140	.0138	.0134	.0133	.0132	.0132	.0132
.75	.0086	.0083	.0078	.0077	.0074	.0073	.0072	.0072	.0072
.80	.0045	.0043	.0040	.0038	.0036	.0035	.0035	.0035	.0035
.85	.0022	.0020	.0018	.0017	.0015	.0015	.0014	.0014	.0014
.90	.0009	.0008	.0007	.0006	.0005	.0005	.0005	.0005	.0005
.95	.0003	.0003	.0002	.0002	.0001	.0001	.0001	.0001	.0001
1.00	.0001	.0001	.0001	.0000	.0000	.0000	.0000	.0000	.0000
1.05	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 7. Continued

	E_o (MeV)								
z/r_o	1.0000	1.5000	2.0000	3.0000	4.0000	5.0000	6.0000	8.0000	10.0000
.00	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000	.5000
.05	.4098	.4077	.4063	.4051	.4045	.4041	.4039	.4039	.4041
.10	.3465	.3436	.3419	.3403	.3396	.3393	.3391	.3392	.3395
.15	.2937	.2906	.2887	.2871	.2865	.2863	.2862	.2866	.2871
.20	.2479	.2449	.2431	.2418	.2414	.2414	.2414	.2420	.2427
.25	.2077	.2050	.2034	.2025	.2023	.2025	.2027	.2036	.2046
.30	.1720	.1697	.1685	.1681	.1682	.1687	.1691	.1701	.1713
.35	.1403	.1386	.1378	.1378	.1383	.1390	.1395	.1408	.1423
.40	.1122	.1111	.1107	.1112	.1120	.1130	.1137	.1152	.1168
.45	.0876	.0869	.0869	.0879	.0890	.0902	.0910	.0928	.0946
.50	.0662	.0660	.0664	.0677	.0690	.0704	.0713	.0733	.0752
.55	.0482	.0484	.0489	.0505	.0519	.0533	.0544	.0564	.0584
.60	.0335	.0338	.0345	.0361	.0376	.0389	.0400	.0421	.0441
.65	.0219	.0223	.0230	.0245	.0259	.0271	.0281	.0301	.0320
.70	.0133	.0137	.0143	.0155	.0167	.0178	.0187	.0205	.0222
.75	.0073	.0076	.0081	.0090	.0099	.0108	.0115	.0130	.0144
.80	.0036	.0037	.0040	.0047	.0053	.0059	.0064	.0075	.0086
.85	.0015	.0016	.0018	.0021	.0025	.0028	.0032	.0039	.0046
.90	.0005	.0005	.0006	.0008	.0009	.0011	.0013	.0017	.0021
.95	.0001	.0001	.0002	.0002	.0003	.0004	.0005	.0006	.0008
1.00	.0000	.0000	.0000	.0000	.0001	.0001	.0001	.0002	.0002
1.05	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0001
1.10	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.15	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000
1.20	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000	.0000

Table 7. Continued

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15. SUPPLEMENTARY NOTES

16. ABSTRACT (A 200-word or less factual summary of most significant information. If document includes a significant bibliography or literature survey, mention it here.)

A calculation has been made of the spatial distribution of absorbed dose in a water medium around monoenergetic point-isotropic electron sources. The calculation takes into account angular deflections and energy-loss straggling due to multiple Coulomb scattering by atoms and orbital electrons; it also includes the transport of energy by secondary bremsstrahlung. The results are presented in the form of scaled point kernels for 36 source energies between 10 MeV and 0.5 keV. The scaling is done by expressing all distances in units of the electron mean range, and makes possible easy interpolation to any source energy in the interval covered. In order to illustrate the use of the point kernels, applications are made to two problems arising in beta-ray dosimetry. The first problem pertains to the self-absorption of energy in spherical source regions. The other problem concerns the absorbed-dose distribution as a function of the distance from a semi-infinite uniform source region.

17. KEY WORDS (Alphabetical order, separated by semicolons)

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